Higher-Order Evaluation of Dipole Moments of a Small Circular Disk*

Bethe[1] was the first to calculate the induced electric and magnetic dipole moments if a plane wave falls upon a circular hole in an infinite plane screen. The corresponding complementary problem of a circular disk can then be solved using Babinet's principle. In some recent experimental work on circular disks in a rectangular waveguide[2] large discrepancies between experimental and theoretical results were found when Bethe's theory was used particularly for disks, where $ka > 0.5$ (a = radius of the disks, $k = 2\pi/\lambda$). Bouwkamp[3] has shown that Bethe's results can be obtained by using a first order approximation for the surface current distribution on the disk. Following the same procedures,[3,4] a sixth order approximation in (ka) for normal incidence and a third order approximation for oblique incidence has been found. The detailed results will be presented in a future paper. Here only the expressions for the electric and magnetic dipole moments will be given.

NORMAL INCIDENCE

Electric dipole moment

$$P = \frac{16}{3}a^3\left(1 + \frac{8}{15}(ka)^2 - j\frac{8}{9\pi}(ka)^3 + \frac{16}{105}(ka)^4 - j\frac{176}{225\pi}(ka)^5\right)\epsilon_0 E_{\tan}^{i}(0,0,0).$$

Magnetic dipole moment

$$M = 0.$$

OBLIQUE INCIDENCE

Electric dipole moment in plane of incidence.

$$P = \frac{16}{3}a^3\left(1 + \frac{8}{15}(ka)^2 - \frac{1}{10}\sin^2\theta(ka)^2\right)\epsilon_0 E_{\tan}^{i}(0, 0, 0).$$

Magnetic dipole moment

$$M = -\frac{8}{3}a^3\left(1 - \frac{2}{10}(ka)^2 - \frac{1}{10}\sin^2\theta(ka)^2 + j\frac{4}{9\pi}(ka)^3\right)H_n^{i}(0, 0, 0).$$

$E_{\tan}^{i}(0, 0, 0)$ = tangential component of electric field at the center of the disk in plane of incidence.

$H_n^{i}(0, 0, 0)$ = normal component of magnetic field at the center of the disk.

a = radius of disk.

θ = angle between the direction of incidence of plane wave and direction of normal to the disk.

From the current distribution on the disk higher order multipole moments can easily be calculated.

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[1] H. A. Bethe, "Theory of diffraction by small holes," *Phys. Rev.*, vol. 66, pp. 163–182; February, 1944.

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[3] C. J. Bouwkamp, "A Critique of Some Recent Developments in Diffraction Theory," New York University, N. Y., Res. Rept. No. EM-50, pp. 44–58; 1953.

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Properties of Symmetric Hybrid Waveguide Junctions*

In his letter[1] J. M. Smith states that in a fully symmetric lossless hybrid where all arms are matched, the amplitudes of the waves traveling in the reverse direction in the main and auxiliary guides are equal. To allow for the existence of the reflected waves this theorem should read:

> "In a slightly imperfect symmetric hybrid with matched loads on the output ports the amplitude of the reflected wave at the input port is equal to the amplitude of the wave at the isolated port."

The proof of this theorem appeared first in the appendix of the paper by H. J. Riblet describing the short slot hybrid.[2]

Although the phase quadrature property of the perfect symmetric hybrid is well known it is not obvious how the 90° phase division varies over a frequency band for a practical imperfect hybrid. The answer to this problem was also given by Riblet,[2] but the result was not well known for a number of years because the result was subsidiary to the main argument of his paper. A more recent paper[3] reviews all the properties of the 90° hybrid junction, and includes the proof of the theorem stated above [see Levy[3] (16)]. The deviation $\Delta\theta$ of the hybrid quadrature property from the ideal 90° is given as a function of the isolation I db by means of the formula

$$\cos\Delta\theta = 2/\text{antilog}\,(I/10). \qquad [\text{Levy}^3\ (19).]$$

This formula reveals the remarkable broadband property of the quadrature phase division, which deviates by only 1.1° from the ideal 90° when the isolation has fallen to 20 db. This is in contrast to the case of 180° ring hybrids, where the phase division is a comparatively rapidly varying function of the isolation.[3]

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[2] H. J. Riblet, "The short-slot hybrid junction," PROC. IRE, vol. 40, pp. 180–184; February, 1952.

[3] R. Levy, "Hybrid junctions—frequency characteristics as phase dividers and duplexers," *Electronic Radio Eng.*, vol. 36, pp. 308–312; August, 1959